

MAGDALENA RUCKA

# GUIDED WAVE PROPAGATION IN STRUCTURES

MODELLING, EXPERIMENTAL STUDIES  
AND APPLICATION TO DAMAGE DETECTION

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*Romuald Szymkiewicz*

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*Janusz T. Cieśliński*

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*Jerzy M. Sawicki*

RECENZENCI

*Wojciech Gilewski*

*Tomasz Mikulski*

PROJEKT OKŁADKI

*Jolanta Cieślawska*

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# LIST OF SYMBOLS AND ABBREVIATIONS

## Symbols

$A$	– cross-sectional area
$A_0, A_1, A_2, \dots$	– antisymmetric Lamb modes
$\mathbf{b}$	– vector of inertia forces
$b$	– width of a structure
$B$	– surface area
$\mathbf{B}$	– strain-displacement matrix
$c_g$	– group velocity
$c_L$	– longitudinal wave speed in a plate
$c_o$	– thin-rod velocity
$c_p$	– phase velocity (wave speed)
$c_P$	– speed of P wave
$c_R$	– speed of R wave
$c_S$	– speed of S wave
$\mathbf{c}$	– vector of damping forces
$\mathbf{C}$	– structure damping matrix
$\mathbf{C}_{(e)}$	– element damping matrix in local element coordinates
$\bar{\mathbf{C}}_{(e)}$	– element damping matrix in global element coordinates
$d$	– half of plate thickness
$D$	– plate flexural rigidity
$\mathbf{D}$	– differential operator matrix
$E$	– modulus of elasticity
$\mathbf{E}$	– stress-strain matrix
$f$	– frequency
$f_x, f_y, f_z$	– distributed loads
$\mathbf{f}$	– vector of distributed loads
$\mathbf{f}_b$	– vector of body forces
$\mathbf{f}_s$	– vector of surface forces
$G$	– shear modulus
$h$	– thickness of a plate
$\mathbf{H}$	– shape function matrix
$i$	– imaginary unit
$I$	– moment of inertia
$J$	– Jacobian
$\mathbf{J}$	– Jacobian matrix
$J_o$	– polar moment of inertia
$k$	– wavenumber
$\mathbf{K}$	– structure stiffness matrix
$\mathbf{K}_{(e)}$	– element stiffness matrix in local element coordinates
$\bar{\mathbf{K}}_{(e)}$	– element stiffness matrix in global element coordinates
$K_L$	– adjustable parameter in the Love theory

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$K_1^{M-H}$ , $K_2^{M-H}$	– adjustable parameters in the Mindlin-Herrmann theory
$K_1^{Tim}$ , $K_2^{Tim}$	– adjustable parameters in the Timoshenko theory
$L$	– length of a structure
$L_e$	– effective length of finite element
$L_N$	– Lobatto polynomial of order $N$
$\mathbf{M}$	– structure mass matrix
$\mathbf{M}_{(e)}$	– element mass matrix in local element coordinates
$\bar{\mathbf{M}}_{(e)}$	– element mass matrix in global element coordinates
$n$	– number of element interpolation nodes
$n_{el}$	– number of elements
$n_r, n_s$	– number of integration points
$N$	– degree of interpolation polynomial
$p$	– excitation force signal (external force)
$p_V$	– excitation voltage signal
$\mathbf{p}$	– vector of external forces
$\mathbf{p}_{(e)}$	– vector of element external forces in local element coordinates
$\bar{\mathbf{p}}_{(e)}$	– vector of element external forces in global element coordinates
$P_N$	– Legendre polynomial of order $N$
$\mathbf{q}$	– vector of nodal displacements
$\mathbf{q}_{(e)}$	– vector of element nodal displacements
$\mathbf{r}$	– vector of internal forces
$\mathbf{r}_{(e)}$	– vector of element internal forces
$S_0, S_1, S_2, \dots$	– symmetric Lamb modes
$t$	– time
$T$	– kinetic energy
$\mathbf{T}$	– transformation matrix from local to global coordinates
$u_x, u_y, u_z$	– translational displacements
$\mathbf{u}$	– vector of displacements
$U$	– potential energy
$v$	– velocity of vibrations
$W_{\text{ext}}$	– work of external forces
$x, y, z$	– Cartesian coordinates
$\beta, \gamma$	– parameters in the Newmark method
$\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$	– shear strains
$\delta_{ij}$	– Kronecker delta
$\delta\mathbf{u}$	– vector of virtual displacements
$\delta W$	– virtual work
$\delta W_{\text{ext}}$	– virtual external work
$\delta W_{\text{damp}}$	– virtual damping work
$\delta W_{\text{int}}$	– virtual internal work
$\delta W_{\text{kin}}$	– virtual kinetic work
$\delta\boldsymbol{\varepsilon}$	– vector of virtual strains
$\Delta t$	– time step
$\Delta t_{cr}$	– critical time step
$\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}$	– normal strains
$\varepsilon_t$	– transverse strain

$\varepsilon$	– vector of strains
$\eta_d$	– damping property parameter
$\kappa$	– adjustable parameter in the Mindlin theory
$\kappa_{KM}$	– adjustable parameter in the Kane-Mindlin theory
$\lambda$	– wavelength
$\Lambda, G$	– Lamé constants of elasticity
$\mu$	– mass density matrix
$\nu$	– Poisson's ratio
$\xi, \eta$	– natural (parent) coordinate system
$\rho$	– mass density
$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$	– normal stresses
$\sigma_{xy}, \sigma_{xz}, \sigma_{yz}$	– shear stresses
$\varphi, \psi_x, \psi_y$	– rotations
$\psi$	– lateral contraction
$\omega$	– circular frequency
$\omega_c$	– cut-off frequency

## Abbreviations

1-D	– one-dimensional
2-D	– two-dimensional
3-D	– three-dimensional
FEM	– finite element method
FFT	– fast Fourier transform
GLL	– Gauss-Lobatto-Legendre
NDE	– nondestructive evaluation
NDT	– nondestructive testing
PZT	– lead zirconate titanate
P wave	– primary, pressure, compressional, extensional, dilatational, irrotational, axial, longitudinal wave
R wave	– Rayleigh wave
SEM	– spectral element method
SFEM	– spectral finite element method
SHM	– structural health monitoring
S wave	– secondary, shear, distortional, rotational, transverse wave
SH wave	– shear horizontal wave
SV wave	– shear vertical wave





## Chapter 1

# INTRODUCTION

### 1.1. Damage detection in engineering structures

Engineering structures undergo gradual destruction in the course of time as a result of static and dynamic loading, temperature, humidity, wind or corrosive factors. Under the influence of these environmental and operating conditions, structures are subjected to fatigue, corrosion, creep and wear. Damage in structural elements may lead to improper operation of any engineering object and it can be a potential threat of financial burden, environmental contamination as well as human lives. After Adams (2007), *damage* is defined here as a permanent change in the mechanical state of a structural material or component that could potentially affect its performance. *Damage detection* is termed as identification of defects and their locations. Damage detection can extend the service life of structures, improve reliability and safety, reduce maintenance costs or even prevent a catastrophic failure. Therefore, the ability to detect structural damage at the earliest possible stage has been of great interest to civil, mechanical and aerospace engineering communities.

The process of assessing the current damage state of a structural material or component without accelerating the damage is termed as nondestructive evaluation (NDE) (Adams 2007). The most common method of a nondestructive assessment of the structure integrity is a periodic visual inspection, mandatory for important structures, for example bridges, which are regularly controlled by experienced engineers. Damage detection can be facilitated by nondestructive testing (NDT) which is the offline implementation of NDE methodologies (e.g. Brunarski and Runkiewicz 1983, Runkiewicz 1999). There are many methods for NDT including radiography (e.g. Lashkia 2001, Ghose and Kankane 2008), acoustic emission (e.g. Rogers 2005, Rahman et al. 2009), infrared thermography (e.g. Clark et al. 2003), ultrasonic testing (e.g. Yeih and Huang 1998, Hoła and Schabowicz 2005), impact-echo techniques (e.g. Lin et al. 2004, Hoła et al. 2009, Rucka and Wilde 2010) or eddy current methods (e.g. Gros 1995). Such diagnostic methods can be effectively applied to damage detection in a few known a priori areas in a structure; however, they can be laborious in searching of potential damage in the whole engineering object.

Further development in local NDT methods leads to so-called structural health monitoring (SHM), which is the online implementation of NDE. Vibration-based and wave propagation methods play a significant role in SHM strategies of dynamics-based global damage detection techniques, where the location of damage is not known. Vibration-based damage detection methods make use of dynamic characteristics of structures (e.g. Dimarogonas 1996, Salawu 1997, Doebling et al. 1998, Ren and Roeck 2002a, 2002b, Uhl 2005, Wilde 2008, Kawecki and Stypuła 2009, Tomaszewska 2010). Recently, many damage detection methods based on structural vibrations, especially combined with genetic algorithms (Kokot and Zembaty 2008, 2009), artificial neural networks (Waszczyszyn and Ziemiański 2001, Kuźniar and Waszczyszyn 2002), modal filters (Mendrok and Uhl 2010), virtual distortion methods (Świercz et al. 2008) or wavelet analysis (Knitter-Piątkowska et al. 2006, Ziopaja et al. 2006, Rucka and Wilde 2006, 2007) have been developed. The

second group of damage detection methods based on dynamics are guided wave propagation techniques, which are the subject of this study. Wave propagation is an extension of the NDT wave testing from the local to global approach of sending and sensing waves. The passage of waves through material thickness is extended to methods based on the long-range guided wave propagation along the structure to inspect large areas rapidly.

## 1.2. Guided wave application in damage detection

Guided wave-based damage detection methods have been dynamically developed over last years. The term *guided wave propagation* refers to wave propagation in bounded media. In the guided wave propagation approach, a structure can be perceived as a waveguide, which directs the wave energy along its length. This technique utilizes excitation of high frequencies (of order a few dozen kHz to a few hundred kHz) in the form of an impulse or a wave packet. Ultrasonic excitation causes that waves are reflected and modes are converted inside a structure, and after some travel superposition causes formation of guided wave modes (Rose 1999).

Since guided waves have the ability of propagation over long distances with a little amplitude loss, they are very suitable for inspecting large structures. Guided Lamb and Rayleigh waves, named after their investigators, are frequently used for damage detection purposes. The Rayleigh waves, discovered by Lord Rayleigh (1885), can propagate in solids containing a free surface. The Rayleigh waves, called also guided-surface waves, travel close to the free surface with very little penetration in the depth, therefore these waves are particularly suitable for detection of surface defects. The Lamb waves, described by Lamb (1917), are guided waves propagating in solid plates with free boundaries. The Lamb wave technique enables to find internal, as well as surface defects, because Lamb waves produce stresses throughout the plate thickness and the entire thickness of the plate is interrogated (Giurgiutiu 2008). Guided waves can also exist in other types of thin-walled structures such as bars, shells and tubes. Other types of guided waves are Love waves travelling in layered materials and Stonley waves occurring at the interface between two media.

Techniques of using Lamb waves for ultrasonic inspection were patented by Firestone and Ling (1951). Their invention was devoted to the method and means for generating and utilizing Lamb waves. Intensive experimental investigations with the use of Lamb waves were undertaken from the early 1960s. Worlton (1961) presented an experimental study on Lamb waves excited in a plate submerged in water. He discussed characteristics of the various modes in the light of potential nondestructive testing applications. In the same year, Grigsby and Tajchman (1961) described properties of Lamb wave propagation which are relevant to possible nondestructive testing application. An experiment was conducted on a steel plate with an artificial flaw using transmitting and receiving ultrasonic transducers. Thompson et al. (1972) developed and fabricated non-contact electromagnetic transducers that enabled to launch and detect ultrasonic flexural Lamb waves of frequency 130 kHz in gas pipelines. Victorov (1976) in his book described experimental research with the use of Rayleigh and Lamb waves for damage detection. He discussed methods for generating and detecting guided waves. Alleyne and Cawley (1992) presented a study of the interaction of Lamb waves with a variety of defects simulated by notches. The finite element results were checked experimentally on a steel 3 mm plate using two conventional wideband ultrasonic immersion transducers and the excitation was in the form of a tone burst modified by a

Hanning window function. Since the 1990s the idea of application of guided waves to damage detection has been followed by many research groups, and it successfully is used in a lot of practical applications, especially in plates (e.g. Giurgiutiu and Bao 2004, Yu and Giurgiutiu 2008), pipes (e.g. Cawley and Alleyne 1996, Lowe et al. 1998, Demma et al. 2004, Rose et al. 2009), rails (e.g. Rose et al. 2004, Lee et al. 2009), composites (e.g. Su et al. 2002, 2009) or aircraft structures (e.g. Dalton et al. 2001, Giurgiutiu et al. 2004).

### 1.3. Aim and scope of study

The purpose of this research is to conduct detailed experimental and numerical investigations on ultrasonic guided wave propagation in steel structures. The particular aims of the present study are:

- modelling of wave propagation phenomenon in structural elements undergoing dispersion effects,
- developing of numerical models for wave propagation,
- systematic experimental verification of the developed numerical models,
- application of the guided wave-based technique to damage detection.

The numerical simulations of wave propagation are performed by the time domain Legendre spectral element method. To model longitudinal, as well as flexural wave propagation taking lateral deformations, shear deformations and rotational inertia effects into consideration, special spectral elements based on higher order theories are formulated, in particular the frame spectral element based on the Mindlin-Herrmann rod theory and the Timoshenko beam theory, as well as the extensional plate element based on the Kane-Mindlin theory and the bending plate element based on the Mindlin theory.

An essential part of the study is devoted to experimental investigations of wave propagation. Longitudinal and flexural waves are excited by means of a piezoelectric actuator and propagating Lamb waves are sensed by a scanning laser vibrometer. A special emphasis is focused on damage detection aspects. Steel structures with discontinuity of material and cross-section are analysed and tested. As a result, this study discusses in detail the possibility of damage detection in bars, frames and plates and it compares the usefulness of longitudinal and flexural waves in nondestructive damage detection.

The content is organized as follows. **Chapter 1** reviews structural health monitoring methods and describes previous researches on the application of guided waves in damage detection. The aim and scope of the study are also presented.

**Chapter 2** describes elastic wave propagation in structural elements. Several models of rods, beams and plates providing approximated description of wave motion have been derived. The necessity of using higher order theories when analysing waves of ultrasonic frequencies is demonstrated.

The formulation of the spectral element method is introduced in **Chapter 3**. The development of time domain spectral elements for a rod, beam, frame, as well as extensional and bending plates is carried out.

In **Chapter 4**, longitudinal, as well as flexural wave propagation in a bar is investigated both experimentally and numerically. In particular, detection of damage in various forms of discontinuity of cross-section and material is considered by analysing wave speeds and time of reflections in guided wave response signals.

**Chapter 5** deals with the mode conversion occurring during longitudinal and flexural wave propagation in planar frames. Three types of frames, namely an L-frame, a T-frame and a portal frame are analysed. Guidelines for SHM systems dedicated for the considered frames concerning the required number of actuators and measurement points are formulated.

In **Chapter 6**, the numerical and experimental studies of Lamb wave propagation in a steel plate are presented. Detection of damage in the form of rectangular surface notch is considered by analysing surface vibration data in the form of A-scans (waveform data plotted as a function of time), B-scans (time-position scans) and C-scans (two-dimensional plane views at selected time instants).

Final remarks and plans for future research are presented in **Chapter 7**.

The idea of the work was initiated by the studies performed by prof. dr hab. K. Wilde, prof. dr hab. J. Chróścielewski, dr W. Witkowski and the author of the work within the confines of the project of Polish Ministry of Science and Higher Education: *Multilevel damage detection system in engineering structures*, no. N506 065 31/3149 (Wilde et al. 2009) and some related papers (see References). The research on guided wave propagation in structures was continued by the author of the work resulting in some journal papers (Rucka 2010a, 2010b, 2011) and finally in this monograph.

The original elements – the results of the author’s scientific research – which have not been published in the joint papers are:

- the experimental and numerical analyses of possibility of damage detection in frame structures indicating the minimum number of actuators and measurement points required to monitor a whole frame,
- the experimental and numerical analyses of possibility of damage detection in plates using B-scans and C-scans,
- the systematic construction of spectral element method models for wave propagation analysis with a special emphasis on dispersion effects and the systematic experimental verification of effectiveness of the proposed spectral element method models,
- the development of the time domain spectral elements for longitudinal waves in rods based on the refined Love and Mindlin-Herrmann theories, as well as the frame spectral element based on the Mindlin-Herrmann rod and Timoshenko beam theories,
- the development of the time domain spectral element for in-plane waves in plates based on the higher order Kane-Mindlin theory.